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HEAT-TRANSFER CHARACTERISTICS OF POLYGONAL AND PLATE FINS

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THIS BRIEF paper is concerned with the efficiency of plate-type fins that consist of a continuous sheet pierced by a regularly spaced array of circular tubes; heat is to be extracted from (or added to) the tubes. The physical situation is illustrated in the center sketches of Figs. 1 and 2, which pertain respectively to tubes in square array and in equilateral triangular array. These sketches also show symmetry lines (dashed) which are, in effect, insulated boundaries. Thus, in Fig. 1, the square region that surrounds the tube behaves like a fin with insulated edges. A similar interpretation applies to the hexagonal region contained within the dashed lines in the sketch of Fig. 2. Thus, the results for the plate fin are equally applicable to a fin of polygonal plan form.

An important contribution to the aforementioned problem has been made by Zabronsky.* Specifically, Zabronsky analysed the case of tubes in square array and devised a temperature solution that exactly satisfies the adiabatic condition at the fin tip and that approximately fulfills the prescribed isothermal boundary condition at the fin base. The analysis yields an expression for the fin efficiency in terms of a double summation extending over a doubly infinite range. An illustrative numerical evaluation is carried out for a specific set of physical parameters.

The present investigation employs an altogether different method of analysis that exactly satisfies the isothermal boundary condition at the fin base; the adiabatic boundary condition at the fin tip is fulfilled approximately, but to any desired accuracy. The analysis is carried out here both for the square and equilateral triangular arrays (i.e. square and hexagonal fins). The results are reported graphically in terms of dimensionless parameters that are standard in engineering application.

The analysis is facilitated by reference to the sketches appearing at the right of Figs. 1 and 2. These represent typical elements of fin, the boundaries of which are symmetry lines (i.e. adiabatic) except for the base $r = r_i$. For a fin of thickness $2t$ and thermal conductivity k exchanging heat with an adjacent fluid having a uniform temperature T_∞ and a uniform heat-transfer coefficient h, an energy balance on a fin element yields

$$2kt\left[\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial^2 T}{\partial \phi^2}\right] = 2hr\,(T - T_\infty) \qquad (1)$$

A solution of equation (1) that exactly satisfies the boundary conditions at $r = r_i$, $\phi = 0$, and $\phi = \phi_0$ ($\phi_0 = \pi/6$ for triangular array and $\pi/4$ for square array) is

$$\frac{T - T_\infty}{T_w - T_\infty} = \frac{I_0(\rho)}{I_0(\rho_i)} + \sum_{n=0} C_n \cos j\phi\left[K_j(\rho) - I_j(\rho)\frac{K_j(\rho_i)}{I_j(\rho_i)}\right] \qquad (2)$$

where $j = n\pi/\phi_0$ ($j = 4n$ or $6n$), $\rho = r\sqrt{[h/kt]}$, $\rho_i = r_i\sqrt{[h/kt]}$, and T_w is the temperature at the base surface. The K and I are modified Bessel functions.

To complete the solution, it still remains to find the constants C_n. For this purpose, one applies the adiabatic condition that $\partial T/\partial N = 0$ (N = normal) at the right-hand boundary of the element, $r = s/\cos\phi$. Indeed, if this condition is imposed at p discrete boundary points, then there are generated p linear equations containing the C_n; further, the series is truncated at $n = p - 1$. This gives p linear, inhomogeneous, algebraic equations for p unknown coefficients $C_0, \ldots, C_{p-1}$. This algebraic system has been solved numerically. In all cases, the number of points p was taken sufficiently large to insure that the final results for fin efficiency are accurate to at least 0·1 per cent.

With the temperature solution at hand, the rate of heat transfer Q at the fin base may be determined by application of Fourier's law. Then, a fin efficiency η may be introduced.

$$\eta = Q/Q_{\text{ideal}}, \quad Q_{\text{ideal}} = 2A_s\,h\,(T_w - T_\infty) \qquad (3)$$

wherein Q_{ideal} corresponds to a fin of infinite thermal conductivity and A_s denotes the area of one surface of the fin. In presenting the results, it is convenient to introduce a fictitious outer radius r_0* that corresponds to an annular fin having the same surface area as the polygonal fins under consideration. This is illustrated

* H. ZABRONSKY, Temperature distribution of a heat exchanger using square fins on round tubes, *J. Appl. Mech.* **22**, 119 (1955).

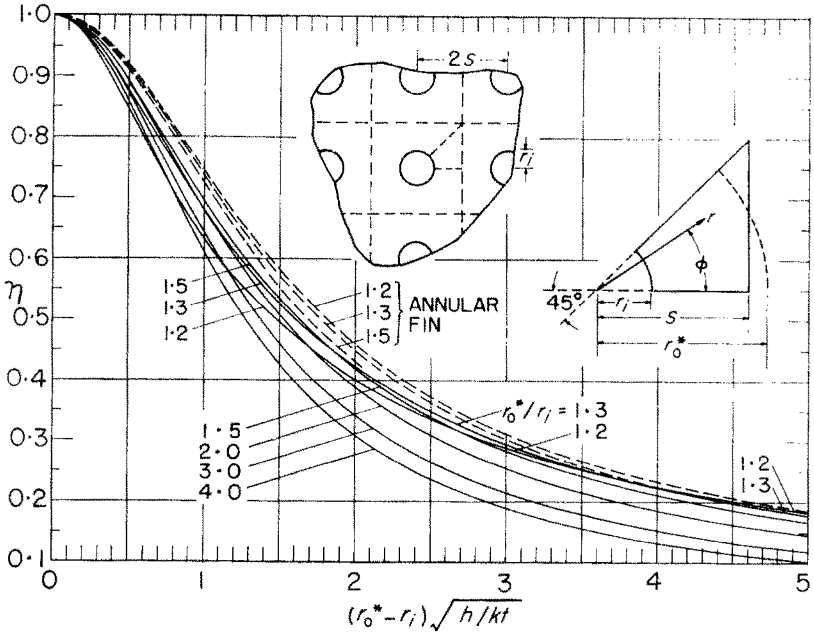

FIG. 1. Efficiency of square fins.

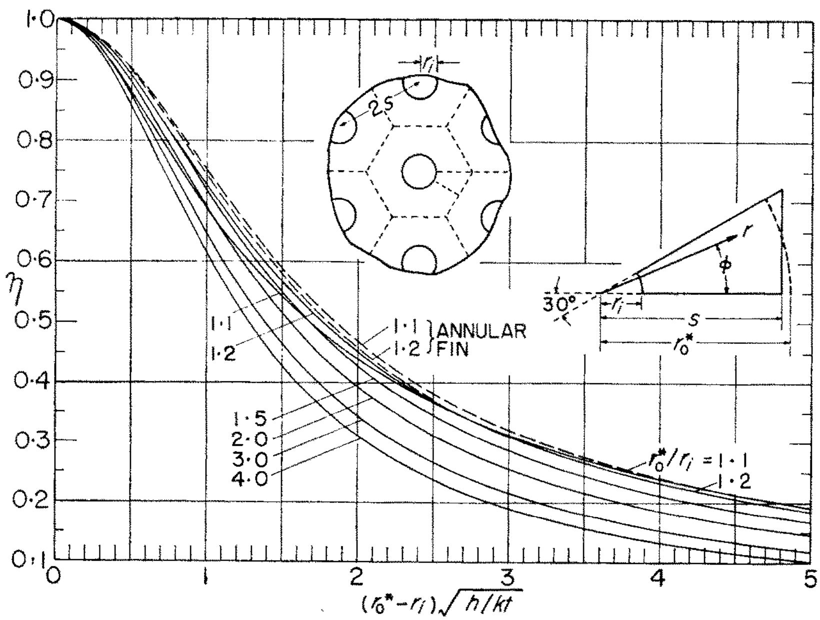

FIG. 2. Efficiency of hexagonal fins.

in the sketches at the right in Figs. 1 and 2. It is readily shown that

$$r_0^* = [2/\sqrt{\pi}]\, s, \quad r_0^* = [2\sqrt{(3)}/\pi]^{1/2}\, s \qquad (4)$$

respectively for the square and the triangular arrays.

The defining equation (3) for η has been evaluated by utilizing the foregoing temperature solution, and the results are plotted as solid lines in Figs. 1 and 2. Also shown in the figures are several dashed lines that represent results for annular fins having radius ratios r_0^*/r_i moderately in excess of unity. Curves for annular fins with larger r_0^*/r_i are not included because they would undermine the clarity of the figures. In general, for given values of the independent parameters r_0^*/r_i and $(r_0^* - r_i)\sqrt{[h/kt]}$, the efficiency of a polygonal fin is lower than that of an annular fin. Moreover, the hexagonal fin is more efficient than the square fin.

Upon considering Fig. 1 in greater detail, it is seen that the deviations between the efficiencies of square and annular fins are most marked for r_0^*/r_i near unity. Indeed, for $r_0^*/r_i = 1{\cdot}2$, deviations in the η curves as large as 18 per cent may occur. For $r_0^*/r_i = 1{\cdot}3$, $1{\cdot}5$, and 2, the maximum deviations are respectively 9, 2·5, and 1·5 per cent. In general, the results for the square and annular fins deviate most in the mid-range of values of the fin conductance parameter $(r_0^* - r_i)\sqrt{[h/kt]}$, in the neighborhood of 1·2. Another interesting feature of Fig. 1 is that the solid curves are not arranged consecutively with r_0^*/r_i. This is to be contrasted with the monotonic arrangement of the annular fin curves as a function of r_0^*/r_i.

Consideration of Fig. 2 shows that the departure of the results for the hexagonal fin from those for the annular fin are smaller than the corresponding departures noted in connection with Fig. 1. The maximum deviations of the η curves for hexagon and annulus are 10 per cent, 2·5 per cent, and 0·5 per cent respectively when $r_0^*/r_i = 1{\cdot}1$, $1{\cdot}2$, and $1{\cdot}5$.

From the foregoing discussion, it is evident that except for radius ratios r_0^*/r_i near unity, the annular fin of equivalent surface area has a heat-transfer efficiency very close to that of a polygonal fin.